NASA TT F-10,300

MEASUREMENT OF THE ATMOSPHERIC TURBULENCE INNER SCALE BETWEEN 80 AND 100 KM

H. Teitelbaum

Translation of "Medida de la escala interna de turbulencia atmosférica entre 80 y 100 km. de altura"
Paper presented at the COSPAR Meeting in Mar del Plata, Argentina, May 1965. CNIE Technical Publication No. 7.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON
OCTOBER 1966

MEASUREMENT OF THE ATMOSPHERIC TURBULENCE INNER SCALE BETWEEN 80 AND 100 KM

H. Teitelbaum

ABSTRACT

Theoretical considerations are presented, introducing the structure function and establishing its form in connection with the so-called inner scale as a function of dissipated energy and kinematic viscosity.

This theory is then used to determine the inner scale of the atmospheric turbulence by means of alkaline clouds; experimental techniques and data are presented.

I. Introduction

In almost all experiments conducted with alkaline clouds in the upper atmosphere, the clouds present a certain property beween 80 and 105 km. In this zone the cloud not only appears to be deformed by winds, but also shows an irregular structure produced by turbulent movements (refs 1 and 2). /1*

We shall show here that information can be obtained on atmospheric turbulence by studying cloud structure, and we shall give the first findings obtained this way.

II. Theory

When a fluid contains a scalar quantity, this quantity is distributed according to the movements of the medium. When fluid passes from a zone with a

*Numbers given in the margin indicate pagination in original foreign text.

fixed concentration to another zone of different concentration, a density gradient is produced which molecular diffusion tends to dissipate. In other words, there is an interaction between turbulent convection and molecular diffusion; the former causes inhomogeneities which the latter destroys.

Let us designate θ as the concentration of scalar quantity. The total inhomogeneity in volume V is defined as :

$$G = \frac{1}{2} \int_V \overline{\theta'^2} \, d\vec{r} \; ; \qquad \theta = \bar{\theta} + \theta'.$$

The variation of G with respect to time is (ref. 3)

$$\frac{\partial G}{\partial t} = \int_V K(\operatorname{grad} \bar{\theta})^2 - D\overline{(\operatorname{grad} \theta')^2} \, d\vec{r}$$

where D = coefficient of molecular diffusion, and

K = coefficient of turbulent diffusion.

When there is equilibrium between the formation and the destruction of inhomogeneities, we have:

$$K(\operatorname{grad} \bar{\theta})^2 = D\overline{(\operatorname{grad} \theta')^2} = N. \qquad (1)$$

N is the quantity of inhomogeneity which is formed in the unit time as a result of turbulent movements, and which dissipates due to molecular diffusion. It plays the same part in the distribution process of a scalar as does energy dissipated in the movement of the fluid. This theory, which has been applied to the study of heat transfer in a turbulent fluid, permits an interpretation to be made of the mixture of gases when the coefficient of mutual diffusion is constant (ref. 4).

Just as for a field of velocities, we can define a structure function as:

$$D_\theta(\vec{r}) = \overline{\left[\theta(\vec{r}_o) - \theta(\vec{r}_o + \vec{r})\right]^2} . \qquad (2)$$

This type of function was first used by Kolmogorof to study locally homogeneous fields where a correlation function cannot be defined. /2

Beginning with dimensional considerations, and bearing in mind the parameters on which the structure function may depend for different ranks of the variable, the following has been established for the function:

$$D(r) = \begin{cases} \frac{1}{3}\frac{N}{D} r^2 & \text{for} \quad r \ll l_o \qquad (3) \\ a \frac{N}{\epsilon^{1/3}} r^{2/3} & \text{for} \quad l_o \ll r \ll L , \qquad (3') \end{cases}$$

where N corresponds to expression(1);

ϵ is the energy dissipated for unit mass and unit time;

L is the distance for which the structure function attains its maximum value; it is also called the turbulent outer scale;

l_o is the abscissa of the intersection of (3) and (3'); it is called the turbulent inner scale; it is the parameter we measured; and

a is a dimensionless constant.

From (3) and (3'), we derive

$$l_o = \left[\frac{(3\,a\,D)^3}{\epsilon}\right]^{\frac{1}{4}} \qquad (4)$$

$$\epsilon = \frac{27\,a^3\,D^3}{l_o^4} . \qquad (5)$$

We will obtain the value of constant a by spectral functions.

Considering that the concentration is an aleatory function of a locally homogeneous field, it can be represented by a Fourier-Stieljes stochastic integral

$$\theta(\vec{r}) = \theta(0) + \int_{-\infty}^{\infty} (1 - e^{i\vec{k}\vec{r}})\, dZ(\vec{k}) .$$

Considering that

$$dZ(\vec{k})\, dZ(\vec{k}) = \Phi(\vec{k})\, \delta(\vec{k} - \vec{K})\, d\vec{k}\, d\vec{k} ,$$

the following results for the structure function

$$D_\theta(\vec{r}) = 2 \int_{-\infty}^{\infty} (1 - \cos \vec{k}\vec{r})\, \Phi(\vec{k})\, d\vec{k} .$$

When the field is isotropic, by integrating in all directions in the space of wave numbers we obtain

$$D_\theta(r) = 8\pi \int_0^{\infty} \left(1 - \frac{\operatorname{sen} kr}{kr}\right) k^2\, \Phi(k)\, dk , \qquad (6)$$

from which we derive: /3

$$N = 4\pi D \int_0^{\infty} k^4\, \Phi(k)\, dk \qquad (7)$$

The form $\Phi(k)$ which satisfies expression (6) with D(r) given by (3') is (ref. 3)

$$\Phi(k) = 0.033\, a\, \frac{N}{\varepsilon^{1/3}}\, k^{-11/3} \qquad (8)$$

For equal coefficients of molecular diffusion and of kinematic viscosity, the energy spectrum and the inhomogeneities take the same form up to a determined value of k from which both decrease rapidly (refs. 5 and 6).

Townsend (ref. 7) showed that the dissipation of energy takes place basically around a wave number expressed as

$$k = \frac{1}{2} \left(\frac{\varepsilon}{\nu^3}\right)^{\frac{1}{4}} . \qquad (9)$$

If we calculate N from expression (7), with Φ (k) given by (8), and integrate to the value of k given by (9), we can calculate the value of a, which is

$$a = 8 . \qquad (10)$$

The structure function of the velocities brings us to the following expression for the inner scale

$$l_o = \left[\frac{(15\, C\, \nu)^3}{\varepsilon}\right]^{1/4} ,$$

where ν is the coefficient of kinematic viscosity,

C is a dimensionless constant.

By another procedure the value of constant C was estimated at 1.6 (ref. 8).

Assuming

$$\left[\frac{(15\, C\, \nu)^3}{\varepsilon}\right]^{1/4} = \left[\frac{(3\, a\, D)^3}{\varepsilon}\right]^{1/4} ,$$

we again obtain a = 8 for the case in which D = ν.

III. Application to alkaline clouds

The sodium cloud is a scalar quantity. Its atomic weight is practically equal to the average molecular weight of atmospheric components at 100 km, in the case of D = ν.

In the case of using trimethyl-aluminum, the substance which becomes visible is aluminum oxide. Its molecular weight is larger; the coefficient of molecular diffusion is therefore smaller.

When $\nu \gg D$, a dissipative zone can be defined in the spectrum which is found beyond the intersection due to viscosity (refs. 5 and 6): /4

$$\left(\frac{\varepsilon}{\nu^3}\right)^{1/4} \ll k \ll \left(\frac{\varepsilon}{D^3}\right)^{1/4}.$$

There is then a distinct form for the spectrum as well as for the structure function. In the case considered, this zone is sufficiently reduced for it to be similar to the case of sodium.

Considered is the measurement of the structure function in alkaline clouds to derive the value of the inner scale of atmospheric turbulence.

We shall obtain the function, except for an unimportant factor, since the use of the logarithmic scale permits us to find l_0 as an abscissa of the point of intersection of two lines of slopes 2 and $\frac{2}{3}$.

IV. Experimental technique

The measurements were made on photographs of the clouds taken with a telescope having a focal distance of 60 cm and a diameter of 50 cm.

A photometer equipped with a notch which can be adjusted for width and height permits us to measure the transparency of the negative on points along the axis of an approximately cylindrical zone of the cloud.

We performed the sensitometry of the film by means of a tube sensitometer.

We must consider the angle formed by the cloud and the focal time of the telescope, as well as the thickness of the former projected on the latter. This makes the function which we measure on the plate

$$\left[\int_0^{a\sec\varphi} \theta\,(x_0, y + x_0 \operatorname{tg}\varphi)\,dy - \int_{x\operatorname{tg}\varphi}^{a\sec\varphi + x\operatorname{tg}\varphi} \theta\,(x_0 + x, y' + x_0 \operatorname{tg}\varphi)\,dy'\right]^2 .$$

We reduced this function to a calculable form and we calculated it for different values of the angle of projection and of l_o. In this way we traced a curve which gives us the relation with the average inner scale measured on the film (designated l'_o and l_o); this permits us to make the necessary corrections (see fig. 1).

The sodium can be seen because of optical resonance; consequently, when the optical thickness is greater than 0.3, multiple diffusion inside the cloud falsifies the results.

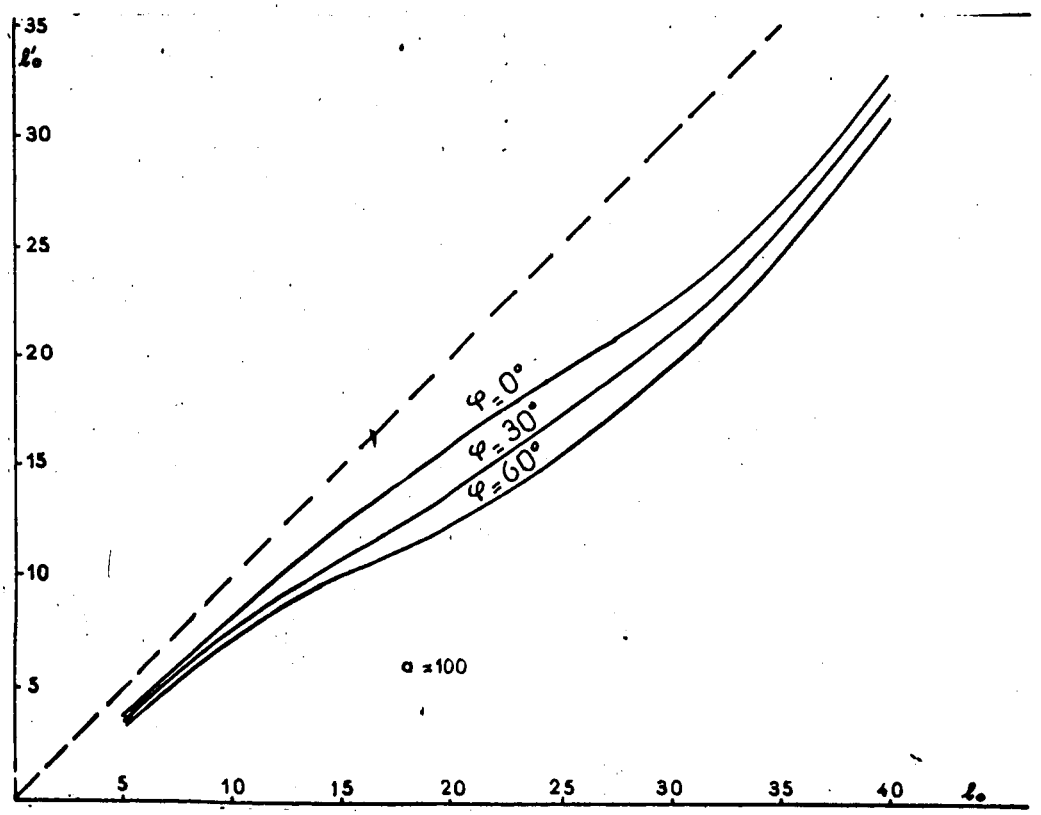

Figure 1.

Our measurements with sodium were made in an absorption-visible zone on the bottom of another higher zone of the same cloud. In this case the effect of multiple diffusion is negligible.

A relative error in estimating the distance between the telescope and the cloud produces the same relative error in l_o. Errors of 20 degrees in the angle of projection cause a 10 percent error in the most unfavorable part of the correction curve. Nevertheless, to know these data precisely enough for photogrammetric restitution of the cloud, we assumed a 15 percent error.

V. Results

For a sodium cloud experiment,

photographs taken of two absorption-observed zones on February 13, 1964 at 6 am, in Hammaguir, were studied. Results indicated that:

$l_o = 320 \pm 50$m at 92km altitude (fig. 2)

$l_o = 470 \pm 70$m at 100km (fig. 3)

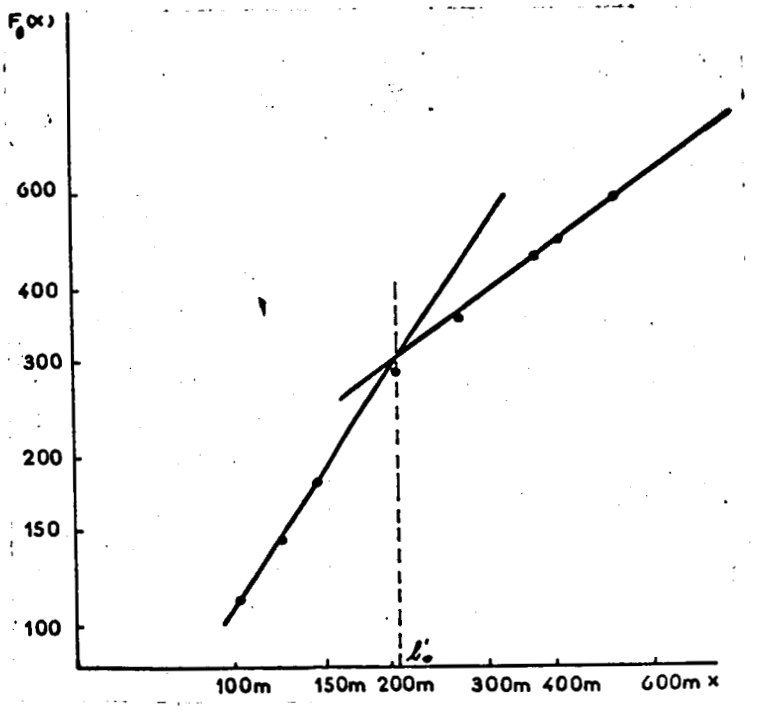

Figure 2

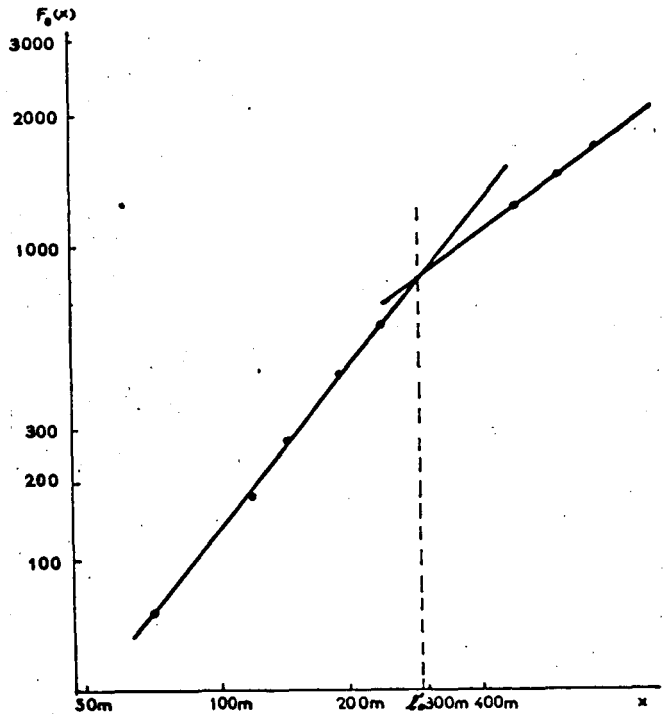

Figure 3

On 13 February 1964, also in Hammaguir, a trimethyl-aluminum cloud was photographed. The result was:

$$l_o = 190 \pm 30\text{m} \quad \text{at } 95\text{km (fig. 4)}$$

We calculated the value of ϵ with the coefficient of kinematic viscosity given for the ARDC model.

This is obviously quite unreliable, in the first place because l_o is raised to the fourth power, and in the second place because ν varies greatly with the altitude. Nonetheless, we feel that these results give an idea of the dissipation.

The sodium experiment gave the following results:

$\epsilon =$ 5 erg gm^{-1} s^{-1} at 92 km.

$\epsilon =$ 100 erg gm^{-1} s^{-1} at 100 km.

For the trimethyl-aluminum, the result is:

$\epsilon =$ 220 erg gm^{-1} s^{-1} at 95 km.

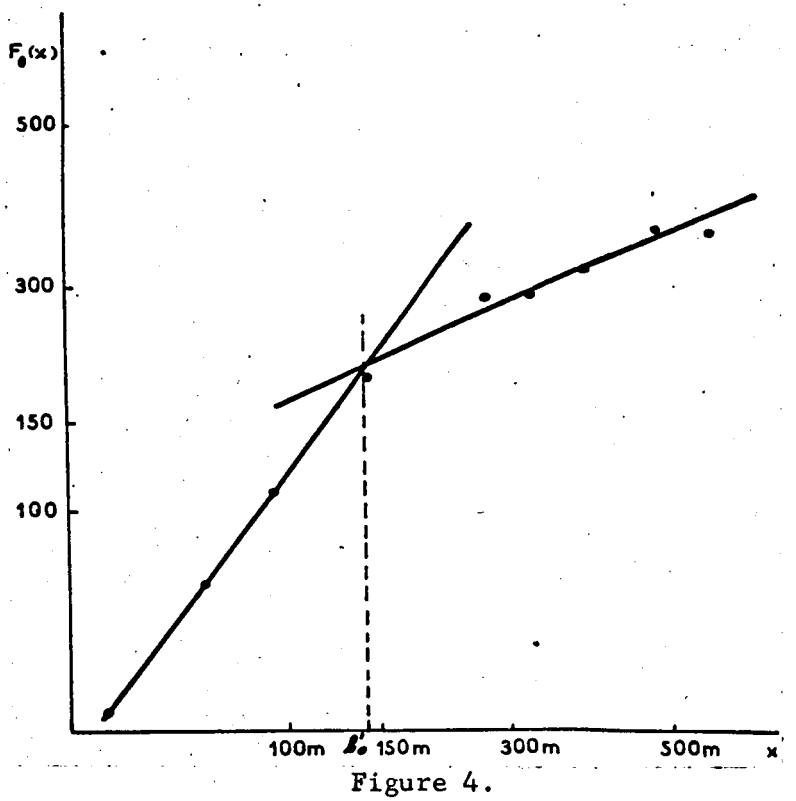

Figure 4.

This study was made in the Service d'Aéronomie de CNRS (France). My grateful appreciation to Professor Blamont, who directed and encouraged me during my research. I also thank the Argentine National Commission on Space Research, which suggested and facilitated my stay in that service.

REFERENCES

1. Blamont, J.E. and De Jager, C. Annales de Geophysique, 17, 134 , 1961.
2. --- and Kallman bijl, H. Space Research, Proceedings of the First International Space Science Symposium, 109 , 1960.
3. Tatarskv, V. E. Wave Propagation in a Turbulent Medium, McGraw-Hill Book Company, 1961.
4. Corrsin, S. J. App. Physics, 22 469 , 1951.
5. Bathelor, G. K. J. Fluid Mech. 5 113, 1959.

6. --- J. Fluid Mech., 5, 134, 1959.

7. Townsend, A. Proc. Roy. Soc. London 208 A, 534, 1951.

8. Agostini, L. and Bass J. The Theories of Turbulence. Scientific and Technical Publications of the Ministry of Air (Les Théories de la Turbulence. Publications Scientifiques et Techniques du Ministère de l'Air).